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Random Discrete Imperfections in Millimeter Waveguide Systems

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***Abstract*—A method has been developed to compute the increase of attenuation due to imperfections of finite length randomly distributed in a link. As a limit for the vanishing length the formulas yield the result for random discontinuities. The approach is quite general and can apply to a circular waveguide link as well as to other cases, where the statistics of the problem are described by the power spectrum of the deformation. The applications presented here show how random spacing of the deformations causes significant modifications on the attenuation results; as a particular case some expressions are found to be in agreement with others previously derived. The results are interesting in determining what random variation in the waveguide lengths is sufficient to avoid a serious frequency dependent effect in the attenuation characteristic of a circular waveguide link.**

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I. INTRODUCTION

IN an overmoded circular waveguide link, coupling may arise between the propagating TE_{01} mode and the other unwanted modes because of many different geometrical imperfections in the guiding structure. Coupling of the TE_{01} mode with the higher order circular electric modes is the more serious instance of such coupling and can occur because of the presence of mirrors [1], diameter discontinuities at the joints, or the manufacturing process. Attenuation peaks, found experimentally [2], can be attributed to this higher order circular electric mode conversion [3]. In fact discontinuities are always present in a circular waveguide link and eventually are causes of coupling. Diameter variations generate higher order TE_{0n} modes, axis tilts, or offset the TE_{1n} and TM_{1n} modes, etc. Great attention has been devoted to the problem of determining the increase of attenuation due to these discrete random imperfections. Here an approach is presented to compute the solution, based on the possibility of considering the discrete imperfections as a limit of continuous deformations of finite extension in space. The relation between discrete and continuous imperfections has been described in [4] and [5] and will not be repeated in this paper. The expected value of the increase of losses can be computed by means of a formula involving the power density spectrum of the deformations. Thus the aim is a derivation of this parameter in a suitable way. In other cases, where the statistics of the problem are described by the spectrum itself (e.g., see some formulas of fiber optics [6]) the same procedure can be useful. With the mathematical model presented here, it is possible to also consider the effect of unequal spacing or other complications of the physical model. In the applications discussed, previously known results are found as a particular case, to emphasize the effect of the random spacing in the case of diameter variation and generation of higher order TE_{0n} modes.

II. Mathematical Model

In this section only diameter and straightness variations will be discussed. In fact, because of the jointing process of the waveguide lengths, other but negligible discontinuities of the section parameters arise. At first, as previously mentioned, the deformations will be considered to be of nonvanishing length and then the results for impulsive δ functions will be derived in the limit. This procedure allows us to compute the expected value of the increase of the attenuation constant α_i due to the ith coupled mode, by means of a convolution (see for example, [7], [8]) in the ζ[1] domain:

$$\alpha_i = \tfrac{1}{2} C_i^2 \int_{-\infty}^{\infty} G(\zeta) W\left(\frac{\Delta\beta_i}{2\pi} - \zeta; \Delta\alpha_i\right) d\zeta \qquad \text{(1a)}$$

$$W(\zeta\Delta\alpha_i) = \frac{2|\Delta\alpha_i|}{|\Delta;\alpha_i|^2 + (2\pi\zeta)^2} \qquad \text{(1b)}$$

where C_i is the coupling coefficient (assumed real) between the TE_{01} mode and the ith coupled mode, $\Delta\alpha_i + j\Delta\beta_i$ is their propagation constant difference, and $G(\zeta)$ is the power density spectrum of the deformation causing the coupling. For small imperfections the total increase of attenuation can be computed by means of this perturbative solution, by adding the contributions due to all the coupled modes for all the deformations (one exception to the possibility of considering only modes coupled to the TE_{01} mode at a time is quoted in [9]).

In two consecutive positions, the deformations may or may not have correlated amplitudes; this depends on the particular cause which produces the deformation itself. For example, in the case of diameter variations, two consecutive waveguide lengths can have different diameters either because of the difference in the diameters of the mandrels used in the manufacturing process (each assumed practically as an ideal cylinder) or because of a purely random deviation

[1] As the functions representing the deformations are dependent on a spatial variable, their Fourier transforms are dependent on a spatial frequency ζ.

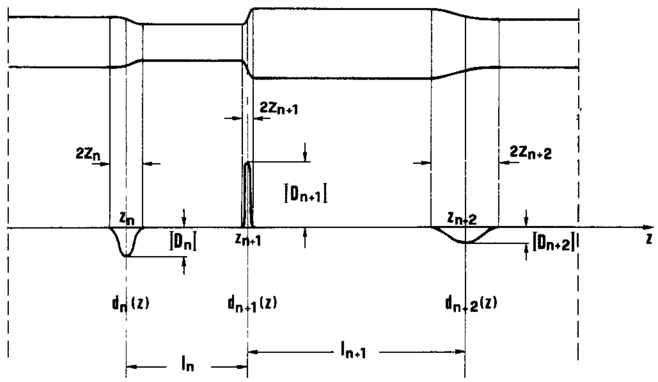

Fig. 1. Example of impulsive deformations in a circular waveguide link.

from the uniform value of a unique mandrel. In the second case one can assume that correlation does not exist between the diameters at the beginning and at the end of a given length of waveguide. In the first case the two values are coincident. The two causes, in fact, act together, but independently, and thus will be taken into account one at a time. A similar thing happens for the tilts or offsets in a straight line, depending on the rigidity of the joints. If the joints are not very rigid, the waveguide lengths follow the wanted path with a series of tilts correlated to each other. Moreover the waveguide lengths can be assumed equal or randomly distributed around a nominal value.

To represent this physical model a suitable mathematical description is a sequence of deformation functions. With reference to Fig. 1, it is assumed that impulsive deformations of a given type are present along the considered waveguide. The variation of their position around a mean value, their amplitude, and their length are assumed as random variables of stationary processes; their shape is assumed constant, as are their main characteristics (e.g., diameter variation, tilt, etc.), and symmetric. Let us denote by $d_n(z)$ the nth deformation function; by D_n its amplitude, by z_n the position of its symmetry axis, and by $2Z_n$ its length. Having defined a normalized variable $s_n = (z - z_n)/Z_n$ each deformation function can be expressed by means of a normalized function $U(s_n)$ nonvanishing only in the $-1 < s_n < 1$ interval:

$$d_n(z) = D_n U(Z_n s_n + z_n). \qquad (2)$$

The deformation of the waveguide link is assumed as a sequence of these random functions $d_n(z)$. Extensive discussion about these impulsive processes can be found in [10], [11], particularly when the random variables z_n are Poisson distributed. Here only a simple way to determine a general expression for the power density spectrum is developed.

With the previous assumptions, the Fourier transform $g(\zeta)$ of the function $U(s_n)$ is

$$g(\zeta) = \int_{-1}^{1} U(s_n) e^{-j2\pi\zeta s_n}\, ds_n. \qquad (3)$$

In a length L of waveguide a particular sequence $f_L^{(k)}(z)$ of the pulse function $d_n(z)$ is present. If $F_L^{(k)}(\zeta)$ denotes the Fourier transform of $f_L^{(k)}(z)$, the power spectrum of the deformation process is

$$G(\zeta) = \lim_{L\to\infty} \frac{1}{L} E(|F_L^{(k)}(\zeta)|^2) \tag{4}$$

where $E(\ )$ denotes the expected value operator. For simplicity, assume $2N+1$ deformations are present, ordered from $-N$ to N, in L. If l_n denotes the difference $z_{n+1} - z_n$ and $l = E(l_n)$, for the ergodicity of the spatial average value, (4) yields

$$G(\zeta) = \frac{1}{l} \lim_{N\to\infty} \frac{1}{(2N+1)} E(|F_N^{(k)}(\zeta)|^2) \tag{5}$$

where the subscript in $F_N^{(k)}$ denotes that $2N+1$ deformations are considered. By means of (2) and (3), (5) yields

$$G(\zeta) = \frac{1}{l} \lim_{N\to\infty} \frac{1}{(2N+1)} E\left(\left| \sum_{-N}^{N} Z_n^{(k)} D_n^{(k)} g(\zeta Z_n^{(k)}) e^{-j2\pi\zeta z_n(k)} \right|^2\right). \tag{6}$$

Finally (6) can be expressed in the following way:

$$G(\zeta) = \frac{1}{l}\left[a + \lim_{N\to\infty} \frac{1}{(2N+1)} \sum_{-N}^{N} \sum_{-N}^{N} b_{n,j}\right] \tag{7}$$

where

$$a = E(|Z_n^{(k)} D_n^{(k)}|^2 |g(\zeta Z_n^{(k)})|^2) \tag{8}$$

$$\begin{aligned} b_{n,j} &= E(D_n^{(k)} D_j^{(k)} Z_n^{(k)} Z_j^{(k)} g(\zeta Z_n^{(k)}) [g(\zeta Z_j^{(k)})]^* \\ &\quad \cdot e^{-j2\pi\zeta(z_n(k) - z_j(k))}), \qquad \text{for } n \neq j \\ &= 0, \qquad \text{for } n = j \end{aligned} \tag{9}$$

and $[\ ]^*$ denotes complex conjugate. Equation (7) is quite general and can be assumed as a starting point for many computations involving power spectra of impulsive deformations.

III. Application to a Circular Waveguide Link

As previously mentioned, the increase of attenuation due to the discontinuities at the joints is wanted for a circular waveguide link. The stated formulas will be used in the limit of the vanishing length for the d_n functions, taking a constant value for the total imperfection [5]. The resulting functions are δ functions of proper area. If, for example, the deformation is expressed by a constant curvature $1/R$ for a length $2Z$, the area of the δ function corresponds to the total angular misalignment $2Z/R$, i.e., the resulting imperfection is a tilt of this value. In the following the d_n functions will be assumed to be of constant length $2Z$ because the limit for vanishing Z_n will be considered.

The first case to be considered is the case of no correlation among the pulses of equal length $2Z$. In this case the imperfections are completely independent. By means of (7)–(9), (1) yields, for $\Delta\alpha_i = 0$,

$$\alpha_i = \frac{1}{2l} C_i^2 E(D_n^{(k)2}) Z^2 \left| g\left(\frac{\Delta\beta_i}{2\pi} Z\right)\right|^2. \tag{10}$$

If, for example, the limit at constant angular misalignment for vanishing Z is considered of short curvature functions, the expression in brackets yields the expected value of the square of the angular variations. In this case, assuming that the expected value of the tilts is zero, (10) yields the expression quoted in [4]. There is no difference, without correlation, between a random distribution of the deformation along the line and a uniform distribution whose period equals the expected value of the random distance.[2]

A second interesting computation is for the diameter variations at a joint, where correlation exists. The random variable z_n can be expressed by

$$z_n = nl + y_n \tag{11}$$

where the random variable v_n has an expected value of zero. Now $ZD_n^{(k)}$ is proportional to the total radius difference $A_n^{(k)}$ between two waveguides:

$$A_n^{(k)} = r_{n+1}^{(k)} - r_n^{(k)}. \tag{12}$$

If a denotes the average radius of the waveguides it is possible to write

$$E(A_n) = 0 \tag{13}$$

$$E((r_n - a)^2) = \sigma^2 \tag{14}$$

$$\begin{aligned} E(A_i A_j) &= 2\sigma^2, \qquad \text{for } i = j \\ &= -\sigma^2, \qquad \text{for } |j-i| = 1 \\ &= 0, \qquad \text{for } |j-i| > 1. \end{aligned} \tag{15}$$

If $U(s_n)$ is chosen so that the proportionality constant between ZD_n and A_n has unitary value, from (8) and (9) it follows that

$$a = 2\sigma^2 |g(\zeta Z)|^2 \tag{16}$$

$$\begin{aligned} b_{n,j} &= -\sigma^2 |g(\zeta Z)|^2 E(e^{-j2\pi\zeta[(n-j)l + v_n - v_j]}), \quad \text{for } |n-j| = 1 \\ &= 0, \qquad \text{for } |n-j| > 1. \end{aligned} \tag{17}$$

If the displacements v_n are supposed not to be correlated, (7) yields

$$G(\zeta) = \frac{2\sigma^2}{l} |g(\zeta Z)|^2 [1 - \cos(2\pi\zeta l) |\theta(v)|^2] \tag{18}$$

where $\theta(v)$ is the characteristic function of v_n in the ζ domain [11].

As an application of (18), the case of the Gaussian pdf of v_n is considered; if, once again, the impulsive function is assumed as a $\delta(s)$ in the limit, (1) and (18) yield α_i for the case of $\Delta\alpha_i = 0$:

$$\alpha_i(\Delta\beta) = \frac{C_i^2 \sigma^2}{l} [1 - \cos(\Delta\beta l) e^{-(\Delta\beta\tau)^2}] \tag{19}$$

where τ^2 is $E(v_n^2)$ and $E(v_n) = 0$.

If (19) is compared with the corresponding expression quoted in [4] for equally spaced conversion ($\tau = 0$), one can

[2] If $\Delta\alpha_i \neq 0$ the convolution (1) must be performed. Nevertheless the conclusions for vanishing Z are valid.

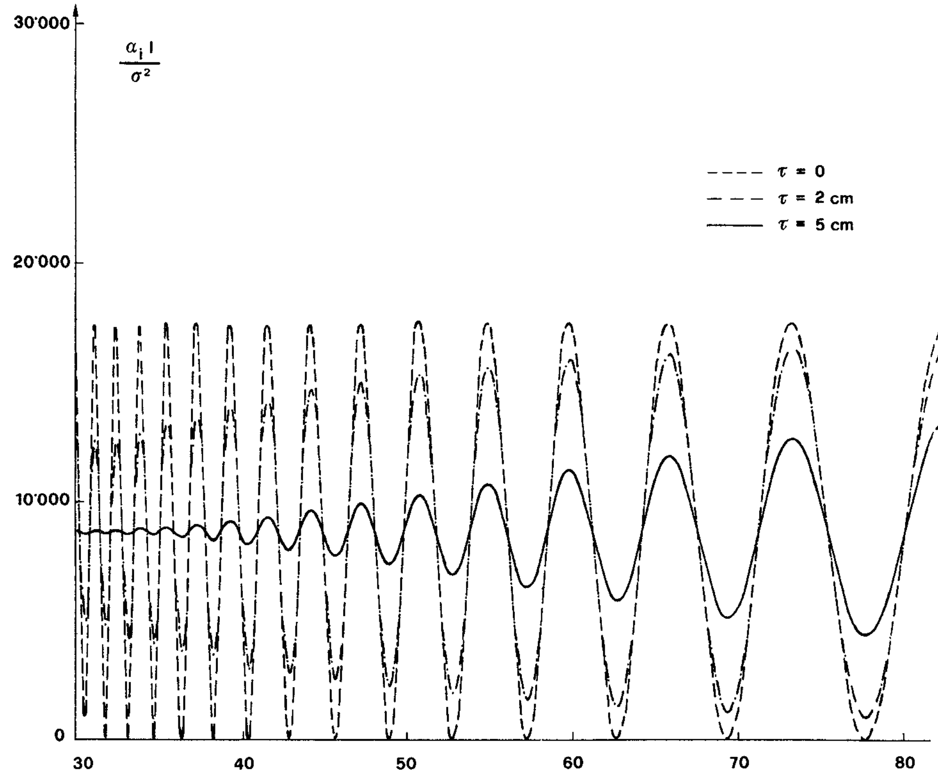

Fig. 2. Normalized expected value of attenuation constant increase $\alpha_i l/\sigma^2$ versus frequency due to the TE_{02} mode, having assumed $\Delta\alpha = 0$ and τ as a parameter, for a 50-mm inner diameter waveguide, manufactured in 5-m lengths.

verify that the presence of a random spacing produces a reduction in the oscillations versus the frequency. For the parameter values quoted, Fig. 2 shows the expected value of the normalized loss increase due to the TE_{02} mode, having neglected the difference in the attenuation constants. The general expression is obtained if the convolution product of (1) is developed for $\Delta\alpha_i \neq 0$; addition of all propagating TE_{0K} modes yields

$$\alpha = \frac{\sigma^2}{l} \sum_{2k}^{m} C_k^2 \Bigg\{ 1 - \frac{e^{(\Delta\alpha_k{}^2 - \Delta\beta_k{}^2)\tau^2}}{2} \cdot \mathrm{Re} \left[\left(e^{-|\Delta\alpha_k| l - 2j\Delta\beta_k|\Delta\alpha_k|\tau^2 + j\Delta\beta_k l} \right) \cdot \left(1 - \mathrm{erf}\left(\tau \left(|\Delta\alpha_k| - \frac{l}{2\tau^2} - j\Delta\beta_k \right) \right) \right) \right] + \mathrm{Re} \left[\left(e^{|\Delta\alpha_k| l + 2j\Delta\beta_k|\Delta\alpha_k|\tau^2 + j\Delta\beta_k l} \right) \cdot \left(1 - \mathrm{erf}\left(\tau \left(|\Delta\alpha_k| + \frac{l}{2\tau^2} + j\Delta\beta_k \right) \right) \right) \right] \Bigg\} \quad (20)$$

where erf (t) represents the error function. Equation (20) yields terms like (19), as a particular solution if $|\Delta\alpha_k| = 0$. One may conclude that random spacing can strongly reduce the oscillation versus the frequency of the attenuation curve and thus improve the behavior of the link. As the main cause of oscillations is the TE_{02} mode, (19), for this mode, it is sufficient to determine a limit value for τ, which reduces the frequency variation of the whole attenuation to the desired level. In practice a variance value of some centimeters, for usual waveguide lengths, is sufficient to obtain a suitable smoothing of the attenuation curve.

In Fig. 3 some plots of $\alpha l/\sigma^2$ versus frequency are presented, for the parameter values quoted in the captions, having considered the $|\Delta\alpha_k|$ values of all propagating circular electric modes in a 50-mm inner diameter copper waveguide. In order to derive from (20) the particular results for equally spaced discontinuities quoted in [4], an asymptotic expression for the erf (t) is useful [12] before taking the limit for vanishing τ.

IV. Conclusion

A general approach to determine the increase of attenuation due to randomly distributed deformations of finite length has been discussed. Some expressions for unequally spaced discontinuities have been derived. Applications to a circular waveguide link are presented in order to emphasize the effect of a random spacing for diameter variations. The same procedure is useful if the case of correlated tilts has to be considered. In this case the coupling coefficients and the attenuation constants of modes have to be computed for the particular waveguide examined: this can be readily performed by means of the program PAGOM [13]. In fact it is not possible to assume, for all the particular waveguide structures, the TE_{1n} and TM_{1n} mode propagation constants referred to an equal diameter copper waveguide as it is allowed, in first approximation, for the circular electric modes TE_{0n}.

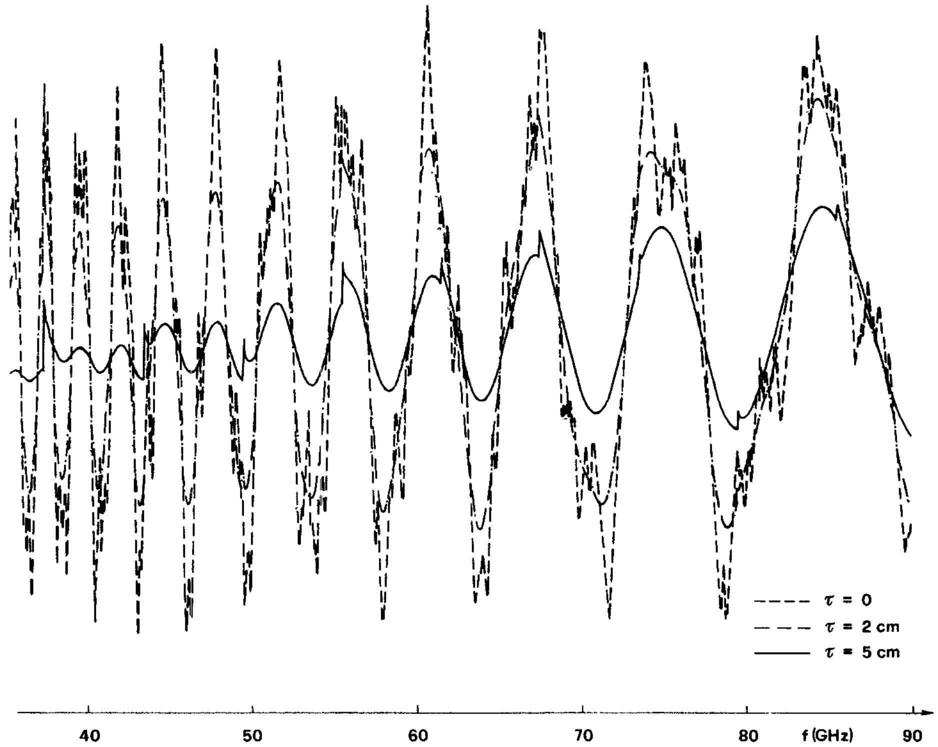

Fig. 3. Normalized expected value of attenuation constant increase $\alpha l/\sigma^2$ versus frequency due to all propagating circular electric modes, having assumed τ as a parameter and the values of $|\Delta\alpha_i|$ of a 50-mm inner diameter copper waveguide, manufactured in 5-m lengths.

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